GU-RET 2016

GAUHATI UNIVERSITY RESEARCH ELIGIBILITY TEST

MATHEMATICAL SCIENCE

Booklet Series : (A)

Invigilator's Name and Signature	
BOOKLET NO.	
OMR SHEET NO.	
ROLL NO.	

TIME : 2 HOURS 20 MINUTES **TOTAL MARKS : 80**

Number of Pages in this Booklet : **16**

Instructions for Candidates

1. Write your Roll No. and OMR Sheet No. in the boxes provided above.
2. This paper consists of two sections : **Section B** with 50 (fifty) multiple choice questions (MCQ) and **Section C** with 6 (six) descriptive questions. Each MCQ has 4 (four) answers, out of which ONLY one is correct. You have to darken the circle (on the OMR Sheet) for the correct answer corresponding to the question given in this booklet.

   Example : (A) (B) (C) (D)

   where (C) is the correct answer. No marks will be given for markings made in this booklet. The descriptive questions in **Section C**, MUST be answered in the space provided in this booklet. **No extra pages will be provided in any case.**
3. Use a BLACK ball point pen in your OMR Sheet.
4. Read the instructions given inside this booklet before attempting to answer any questions.
5. DO NOT write your name, roll no, phone no, or anything, or put any marks anywhere in this booklet, otherwise your candidature will be disqualified.
6. If you are found to resort to any kind of unfair means such as carrying extra material other than pen, pencil, watch, eraser, and scale, or copying from somebody or from external material, your candidature will be disqualified.
7. Use of mobile phones, programmable calculators, log tables or any other tables, wearable smart devices such as smart Android watches or objects of similar nature CAN NOT be used inside the examination hall.
8. At the end of the examination, you have to return this booklet and the OMR Sheet back to the invigilator.
9. There is no negative marks for incorrect answer.

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Section B (50 Marks)

1. Let $L : M_{22} \to M_{22}$ be the linear transformation defined by $L(A) = A^T$, where M_{22} is the space of all 2×2 matrices with real entries, and A^T denotes the transpose of A. Let S and T are respectively given by

$$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$
$$\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

and be ordered bases for M_{22}. If A_1 is the matrix of L with respect to S and T, and if A_2 is the matrix of L with respect to T and S, then Trace(A_1) − Trace(A_2) is

(A) 0
(B) 1
(C) -1
(D) -2

2. Let $L : P_3 \to P_3$ be the linear transformation defined by $L(at^3 + bt^2 + ct + d) = (a - b)t^2 + (c - d)t$, where P_3 is the space of all polynomials of degree at most 3. Then

(A) L is neither one-to-one nor onto
(B) L is one-to-one but not onto
(C) $3t^3 - 3t^2 \in$ Range L but $3t^3 - 3 \in$ Range L
(D) dim(Kernel L) = 1 and dim(Range L) = 3

3. Consider the LPP (Liner Programming Problem) : maximize $2x_1 + 5x_2$ subject to

$$\begin{aligned} x_1 &\leq 4 \\ x_2 &\leq 6 \\ x_1 + x_2 &\leq 8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Once the problem is converted to the standard form, that is, minimize $c^T x$ subject to $Ax = b$, $x \geq 0$, the reduced cost coefficients after applying the Simplex algorithm, are

(A) 1 and 3
(B) 2 and 3
(C) 2 and 5
(D) 3 and 4

4. Consider the Fredholm integral equation

$$y(x) = f(x) + \lambda \int_a^b K(x,t) y(t)\, dt$$

The Resolvent kernel for $K(x,t) = xe^t$, $a = -1$, $b = 1$ is

(A) $\dfrac{xe^t}{e - 2\lambda}$ where $|\lambda| < e/2$
(B) $\dfrac{xe^{t+1}}{2\lambda - e}$ where $|\lambda| < e/2$
(C) $\dfrac{xe^{t+1}}{e - 2\lambda}$ where $|\lambda| < e/2$
(D) $\dfrac{xe^{t-1}}{e - 2\lambda}$ where $|\lambda| < e/2$

5. Consider the following statements from LPP (Linear Programming Problem) in the standard form, that is, minimize $c^T x$ subject to $Ax = b$, $x \geq 0$

1. A basic feasible solution is optimal if and only if the corresponding reduced cost coefficients in the Simplex algorithm are all negative.
2. A basic feasible solution is optimal if and only if the corresponding reduced cost coefficients in the Simplex algorithm are all non-negative.
3. If there exists a feasible solution, then there exists a basic feasible solution.
4. If there exists a feasible solution, then there may or may not exist a basic feasible solution.
5. If there exists an optimal feasible solution, then there exists an optimal basic feasible solution.
6. If there exists an optimal feasible solution, then there may or may not exist an optimal basic feasible solution.

Which of the above are true statements?

(A) 1, 3 and 5
(B) 2, 3 and 5
(C) 1, 4 and 5
(D) 2, 3 and 5

6. Let A be the set of points in the interval $(0,1)$ representing the numbers whose expansion as infinite decimals do not contain the digit 7. Then the measure of A is

(A) 1
(B) 0
(C) 1/2
(D) ∞

7. The function $f(z) = |z|^2$ is

(A) differentiable only at 0
(B) analytic only at 0
(C) not differentiable at 0
(D) None of these

8. Consider the system of linear equations

$$\begin{aligned} 2x + y + z &= 10 \\ 3x + 2y + 3z &= 18 \\ x + 4y + 9z &= 16 \end{aligned}$$

which is of the form $Ax = b$ where A is a 3×3 matrix and b is a 3×1 vector. Consider the following statements

1. The system is consistent
2. The system has infinitely many solutions
3. The system has a unique solution
4. The matrix A associated with the system is rank deficient
5. Matrix A is singular

Then

(A) only 1 and 3 are true
(B) only 1, 2 and 5 are true
(C) only 4 is true
(D) only 1, 2, 4 and 5 are true

9. The solution of the differential equation

$$\frac{dy}{dx} = \frac{1}{2}(1+x)y^2, \quad y(0) = 1 \text{ at } x = 0.1$$

(taking step size $h = 0.1$) by Euler's method and by Modified Euler's method are respectively

(A) 1.04 and 1.0553
(B) 1.042 and 1.0563
(C) 1.05 and 1.0563
(D) 1.05 and 1.0553

10. The integral surface of the equation

$$\frac{dy}{dx} = \frac{1}{2}(1+x)y^2$$

which passes through the curve $z = x^3$, $y = 0$ is

(A) $\dfrac{z}{y+x} = x^2 + y^2$
(B) $\dfrac{z}{y-x} = x^2 + y^2$
(C) $\dfrac{z}{y+x} = x^2 - y^2$
(D) $\dfrac{z}{y-x} = x^2 - y^2$

11. The complete integral of the partial differential equation $zpq = p + q$ is

(A) $\dfrac{z^2}{2} = (C_1+1)x - \left(\dfrac{C_1+1}{C_1}\right)y + C_2$
(B) $\dfrac{z^2}{2} = (C_1+1)x + \left(\dfrac{C_1+1}{C_1}\right)y + C_2$
(C) $\dfrac{z^2}{2} = (C_1-1)x + \left(\dfrac{C_1-1}{C_1}\right)y + C_2$
(D) $\dfrac{z^2}{2} = (C_1-1)x - \left(\dfrac{C_1+1}{C_1}\right)y + C_2$

12. The characteristic curves for the partial differential equation $3u_{xx} - 8u_{xy} + 4u_{yy} = 0$ are families of

(A) straight lines
(B) circles
(C) rectangular hyperbolas
(D) ellipses

13. Consider the Fredholm integral equation

$$y(x) = \cos 3x + \lambda \int_0^{\pi} \cos(x+t)y(t)dt$$

Which of the following is true?

(A) If $\lambda \neq \pm\dfrac{2}{\pi}$ then the given equation has a unique solution
(B) If $\lambda = \dfrac{2}{\pi}$, then the given equation has infinitely many solutions
(C) If $\lambda = -\dfrac{2}{\pi}$, then the given equation has infinite number of solutions
(D) All of the above

14. The functional

$$J = \int_0^{\pi} (y'^2 - y^2)\, dx,\ y(0) = 0,\ y(\pi) = 0$$

has

(A) only one extremal
(B) no extremals
(C) infinite number of extremals
(D) None of the above

15. The extremals of the functional

$$\int_a^b (y^2 + y'^2 - 2y \sin x)\, dx$$

are

(A) $y(x) = Ae^x + Be^{-x} - \frac{1}{2}\sin x$
(B) $y(x) = Ae^x + Be^{-x} + \frac{1}{2}\sin x$
(C) $y(x) = A\sin x + B\cos x - \frac{1}{2}e^x \sin x$
(D) $y(x) = A\sin x + B\cos x + \frac{1}{2}e^{-x}\sin x$

16. Let $f(x_1, x_2) = 5x_1 + 8x_2 + x_1x_2 - x_1^2 - 2x_2^2$. Then the Hessian of f is given by

(A) $\begin{bmatrix} -2 & 1 \\ 1 & 4 \end{bmatrix}$
(B) $\begin{bmatrix} -2 & 1 \\ 1 & -4 \end{bmatrix}$
(C) $\begin{bmatrix} -2 & 1 \\ 1 & -4 \end{bmatrix}$
(D) $\begin{bmatrix} 2 & 1 \\ 1 & -4 \end{bmatrix}$

17. The solution to the Fredholm integral equation

$$y(x) = x + \lambda \int_0^1 (1 - 3xt)y(t)\, dt$$

provided $\lambda \neq 2$ is

(A) $y(x) = x + \frac{\lambda}{\lambda^2 - 4}[2 - (\lambda - 4)x]$
(B) $y(x) = x + \frac{\lambda}{4 - \lambda^2}[2 - (4 - \lambda)x]$
(C) $y(x) = x + \frac{\lambda}{\lambda^2 - 4}[2 - (4 - \lambda)x]$
(D) $y(x) = x + \frac{\lambda}{4 - \lambda^2}[2 - (\lambda - 4)x]$

18. The region in which the PDE $u_{xx} - yu_{xy} + xu_x + yu_y + u = 0$ is hyperbolic is specified by

(A) $y = 0$
(B) $y \neq 0$
(C) $y > 0$
(D) $y < 0$

19. The curve joining two points which generates a surface of revolution of minimum area, when revolved about the x-axis is

(A) a catenary
(B) a cycloid
(C) a straight line
(D) an arc of a circle

20. Forces with components $(2, 0, 0)$, $(-1, 0, 0)$, and $(-1, 0, 0)$ act at $(0, 0, 0), (0, 1, 0)$, and $(0, 0, 1)$. The magnitude of the couple is

(A) $\frac{1}{\sqrt{2}}$
(B) $\sqrt{2}$
(C) 1
(D) 2

21. The forces which may be omitted in forming the equation of virtual work is

(A) weight of a body
(B) acceleration due to gravity
(C) the tension in an inextensible string
(D) None of the above

22. The spectral radius $\rho(A\,B\,A^{-1})$ with

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$$
$$B = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 4 \end{pmatrix}$$

is

(A) 1
(B) 2
(C) 3
(D) 4

23. As a subset of $[0, 1]$ equipped with the usual topology, the Cantor set is

(A) closed but not compact, nowhere dense and uncountable
(B) not closed, dense and countable
(C) closed, dense and uncountable
(D) compact, nowhere dense and uncountable

24. Which of the following statement is false?

(A) Any product of compact spaces is compact
(B) Any product of Hausdorff spaces is Hausdorff
(C) Any product of metrizable spaces is metrizable
(D) Any product of connected spaces is connected

25. For the differential equation $4x^3y'' + 6x^2y' + y = 0$, the point at infinity is

(A) an ordinary point
(B) a regular singular point
(C) an irregular singular point
(D) a critical point

26. The eigenvalues of the Sturm-Liouville system $y'' + \lambda y = 0$, $0 \leq x \leq \pi$, $y(0) = 0$, $y'(\pi) = 0$ are

(A) $\frac{1}{4}n^2$
(B) $\frac{1}{4}(2n-1)^2\pi^2$
(C) $\frac{1}{4}(2n-1)^2$
(D) $\frac{1}{4}n^2\pi^2$

27. The square root of N, where $N > 0$, is calculated by Newton Raphson method. The correct iteration formula is

(A) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{x_n}\right)$
(B) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$
(C) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{2x_n}\right)$
(D) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{2x_n}\right)$

28. Which of the following subsets of R^3 are subspaces? The set of all vectors of the form

(A) $(x, y, 2)$
(B) (x, y, z), where $z = x + y$
(C) (x, y, z), where $z > 0$
(D) All of the above

29. Let G be set of all 3×3 matrices with the addition operation. Let H be set of all 3×3 matrices having trace 0 with addition operation. Then

(A) G is a group but H is not a group
(B) G is a group and H is also a group
(C) G is not a group but H is a group
(D) neither G not H are groups

30. Let $T : R^3 \to R^3$ be the linear transformation

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y \\ y+x \\ x+z \end{pmatrix}$$

Then the Rank(T) is

(A) 0
(B) 1
(C) 2
(D) 3

31. Residue of the function $f(z) = \frac{1}{z+z^2}$ at $z = 0$ is

(A) 0
(B) 1
(C) 1/2
(D) None of these

32. Let X be a random variable with probability density function

$$f(x) = \frac{1}{\pi(1+x^2)}, \; -\infty < x < \infty$$

Then expectation $E(X)$ of X is

(A) 0
(B) 1
(C) 1/2
(D) None of these

33. Let G be a group of order 169. Then G is always

(A) Abelian but not cyclic
(B) cyclic
(C) non-Abelian
(D) None of these

34. The set of all rational numbers in R is

(A) open
(B) closed
(C) neither open nor closed
(D) both open and closed

35. How many different outcomes are there if we roll three dices?

(A) 56
(B) 48
(C) 50
(D) 54

36. The total number of one-one function from $\{1, 2, 3, 4, 5\}$ to $\{1, 2, 3, 4, 5, 6\}$ are

(A) 5
(B) 6
(C) 10
(D) None of these

37. Let $f(x) = x \cos(1/x)$ if $x \neq 0$ and $f(0) = 0$. Then $f(x)$ at $x = 0$ is

(A) not continuous
(B) continuous but not differentiable
(C) differentiable but not twice differentiable
(D) None of these

38. The Laplace transform of Heaviside's unit step function defined by $H(t-c) = 1$ if $t > c$ and $H(t-c) = 0$ if $t < c$ is

(A) $\frac{e^{-sc}}{s}$
(B) $\frac{e^{-s^2c}}{s^2}$
(C) $\frac{e^{-sc}}{s^2}$
(D) None of these

39. Let $f(x) = x^3$. Then f is not uniformly continuos on

(A) $[0, \infty)$
(B) $(0, 1)$
(C) $(0, 1]$
(D) $[0, 1]$

40. Let $x_n = 1 + (-1)^n$. Then $\overline{\lim_{n\to\infty}} x_n$ is equal to

(A) 0
(B) 1
(C) -1
(D) 2

41. The principal value of i^i is equal to

(A) $e^{-\pi/2}$
(B) $e^{-\pi}$
(C) $e^{-2\pi}$
(D) None of these

42. A continuous random variable X has a probability density function $f(x) = 3x^2$, $0 \leq x \leq 1$. The value of a such that $P(X \leq a) = P(X > a)$ is

(A) $1/2$
(B) $1/3$
(C) $(1/2)^{1/3}$
(D) $(1/2)^{1/2}$

43. The total number of roots of the equation $z^7 - 4z^3 + z - 1 = 0$ inside the circle $|z| = 1$

(A) 1
(B) 2
(C) 3
(D) 4

44. The total number of generators of the group $z_3 \times z_{11}$ is

(A) 20
(B) 33
(C) 34
(D) None of these

45. Let $P_3(x)$ be the vector space of polynomials of degree at most 3 in x with real coefficients over R. Let B be the ordered basis $B = \{1, 1+x^2, 1+x, x^3\}$. Then the coordinate matrix of the element $1 + x + x^2 + x^3$ with respect to B is

(A) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

46. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{z^{3n}}{2^n}$$

is

(A) 0
(B) $1/2$
(C) $2^{1/3}$
(D) 2^3

47. The orthogonal trajectories of family of curves $y = ax^2$ are

(A) $\frac{x^2}{2} + \frac{y^2}{1} = C$
(B) $\frac{x^2}{1} + \frac{y^2}{2} = C$
(C) $x^2 - y^2 = C$
(D) $y^2 - x^2 = C$

48. Let $x_n = 5 + 3\cos(n\pi)$ for $n \geq 0$. Then x_n is

(A) convergent
(B) unbounded and oscillatory
(C) diverges to $\pm\infty$
(D) None of these

49. Let $F_n = [1/n, 1 - 1/n]$, $n \geq 3$ be a sequence of closed intervals. Then $\bigcup_{n=1}^{\infty} F_n$ is

(A) $[0, 1]$
(B) $(0, 1]$
(C) $[0, 1)$
(D) $(0, 1)$

50. Let $f(z) = \frac{1 - \cos z}{z^2}$

(A) pole of order 2
(B) removable singularity
(C) essential singularity
(D) None of these

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Section C (30 Marks)

Answer any 5 (five) from the following

1. Represent the function
$$f(z) = \frac{z+1}{z-1}$$
by its Laurent Series in the domain $1 < |z| < \infty$. Find the Maclaurin Series and state where the representation is valid. (Marks : 6)

2. Show that (Marks : 6)
$$\int_0^\infty \frac{dx}{x} = \infty$$

3. Consider the basis $B = (1,1,1), (1,0,-1), (-1,2,3)$ for R^3. Find an orthogonal basis for R^3 using Gram-Schmidt process. (Marks : 6)

4. Consider the Linear Program : (Marks : 6)
$$\begin{aligned} \text{maximize } & 7x_1 + 6x_2 \\ \text{subject to } & 2x_1 + x_2 \leq 3 \\ & x_1 + 4x_2 \leq 4 \\ \text{with } & x_1, x_2 \geq 0 \end{aligned}$$
Convert the above problem to the standard form: minimize $c^T x$ subject to $Ax = b$, $x \geq 0$, and solve it using the Simplex method by forming the canonical tableau.

5. A string of length 3 units (with two ends fixed) is pulled to an initial shape of $\sin x$ and is allowed to vibrate from rest. Find the points x which has zero displacement for all time t. Also, at what times t the displacement is zero at all points x? (Marks : 6)

6. Is every Cauchy sequence of real numbers convergent? Justify. (Marks : 6)

Space for Answers (Section C) : for Questions 1 to 5 (6 pages)

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